II. CONSTITUTIVE RELATIONS

2.1 General Considerations

The term "constitutive relations" is used in a generic
sense to encompass all material properties which must be combined
with the equations of continuity, motion, and energy conserva-
tion to supply a complete set of flow equations. Constitutive
relations in practice usually reduce to an equation of state
relating pressure, volume, and temperature or internal energy.
This simplification is partially enforced by ignorance of other
material relations; it often yields, in addition, quite a good
description of wave propagation over a wide range of parameters.
The equation of state is necessarily accompanied by a statement
about the variation of specific heat with pressure and tempera-
ture. It may on occasion include information about rigidity and
yield or even rate effects. For the present we consider the
equation of state as defined above. These considerations are
themselves useful and they will provide insight into the require-
ments which must be satisfied by more general constitutive rela-
tions. The following remarks are necessarily limited in their
scope. More detailed information will be found in various review
articles (1,2).

2.2 Equations of State for Fluids (3)

A "complete" equation of state for a fluid is a rela-
tion between thermodynamic variables which is sufficient for

3

calculating any thermodynamic parameter of the material, given
two. For example, if specific internal energy, E, is given as
a function of specific entropy, S, and specific volume v,
$E = E(S,v)$, then

$$p = -(\partial e/\partial v)_S$$
$$T = (\partial e/\partial S)_v$$
$$H = E + pv$$
etc.,

where p, T, H are pressure, temperature and specific enthalpy.
On the other hand, if pressure is given as a function of T and
v, as usually occurs, then neither internal energy nor enthalpy
can be calculated without specifying the specific heat.

When partial equations of state are given, as in the
last example, then certain limitations are placed on other
thermodynamic quantities if all are to be compatible. This is
illustrated by the following example. Suppose that specific
heat at constant pressure is known as a function of temperature
and assumed to be a function of temperature alone: $C_p = C_p(T)$.
Then by the following argument we can see that the relation be-
tween p, v, and T must be that of Eq. (2.1) below:

$$(\partial H/\partial T)_p = C_p(T) \quad \text{by definition.} \tag{2.1}$$

If $H = H(p,T)$, then

$$dH = C_p dT + (\partial H/\partial p)_T \, dp$$

or

$$H = \int C_p(T) dT + f(p) .$$